CP-odd Weak Basis Invariants for Neutrino Mass Matrices with a Texture Zero and a Vanishing Minor

S. Dev[*], Shivani Gupta[†] and Radha Raman Gautam[‡]

Department of Physics, Himachal Pradesh University, Shimla 171005, India.

Abstract

We construct the CP-odd weak basis invariants in the flavor basis for all the phenomenologically viable neutrino mass matrices with a texture zero and a vanishing minor and, also, find the necessary and sufficient conditions for CP invariance . We examine the interrelationships between different CP-odd weak basis invariants for these texture structures and investigate their implications for Dirac- and Majorana-type CP violation.

1 Introduction

The evidence for nonvanishing neutrino masses provides a clear signal for physics beyond the Standard Model (SM). In most extensions of the SM, there can be several CP violating phases. The violation of CP symmetry is established in the quark sector and it is natural to expect that CP violation occurs in the lepton sector too. The study of CP violating phases is imperative especially in view of the development of

[*]dev5703@yahoo.com
[†]shiroberts_1980@yahoo.co.in
[‡]gautamrrg@gmail.com

the challenging and rather expensive experimental program to measure CP violation in neutrino oscillations. In the simplest scenario of three generations, there can be one CP phase in the mixing matrix in the leptonic sector. In addition if the neutrinos are Majorana particles, there can be two additional phases. It is possible to work in the parametrization in which all of three CP violating phases are situated in the charged current lepton mixing matrix. Without any loss of generality, one can work in the flavor basis in which the charged lepton mass matrix is diagonal. The neutrino mass matrix in this basis will then contain all the information about CP violation. The search for CP violation in the leptonic sector at low energies is one of the major challenges for experimental neutrino physics. The two Majorana-type CP violating phases will contribute to lepton number violating (LNV) processes like neutrinoless double beta decay while the Dirac-type CP violating phase δ is expected to be measured in the experiments with superbeams and neutrino beams from neutrino factories or indirectly through the area of the unitarity triangles defined for the leptonic sector. Thus, neutrino physics provides an invaluable tool for the investigation of leptonic CP violation at low energies apart from having profound implications for the physics of the early universe. It is not possible to fully reconstruct the neutrino mass matrix from the observations of feasible experiments and it is, thus, natural to employ other theoretical inputs for the reconstruction of neutrino mass matrix. Several proposals have been made in the literature to restrict the possible forms of neutrino mass matrix by reducing the number of free parameters which include the presence of texture zeros [1, 2, 3], hybrid textures [4], vanishing minors [5] and more recently, simultaneous existence of a texture zero and a vanishing minor [6]. However, not all these measures taken to reduce the number of free parameters are weak basis invariant. For example, a texture zero and a vanishing minor in a certain weak basis may not be present at all or may appear at a different place in another weak basis (WB). But two sets of leptonic mass matrices related by WB transformations contain the same physics. Thus, it is of utmost importance to analyse specific flavor models in a basis independent manner. CP-odd WB invariants provide invaluable tools to study CP violation both in the quark and the leptonic sector. The interest in WB invariants stems from the fact that they can be evaluated and analyzed in any conveniently chosen WB and are, thus, particularly suited to the analysis of specific Ansätze for charged leptons and neutrino mass matrices. The CP-odd WB invariants must vanish for CP invariance to hold. Nonvanishing values of any of these

WB invariants would signal CP violation. Low energy CP-odd weak basis invariants for two texture zero neutrino mass matrices have been studied in Ref. [7].

The seesaw mechanism for understanding the scale of neutrino masses is regarded as the prime candidate not only due to its simplicity but also due to its theoretical appeal. In the framework of type I seesaw mechanism [8] the effective Majorana mass matrix M_ν is given by

$$M_\nu = -M_D M_R^{-1} M_D^T \tag{1}$$

where M_D is the Dirac neutrino mass matrix and M_R is the right-handed Majorana mass matrix. It has been noted by many authors [9, 10] that the zeros of the Dirac neutrino mass matrix M_D and the right-handed Majorana mass matrix M_R are the progenitors of zeros in the effective Majorana mass matrix M_ν. Thus, the analysis of zeros in M_D and M_R is more basic than the study of zeros in M_ν. However, the zeros in M_D and M_R may not only show as zeros in effective neutrino mass matrix. Another interesting possibility is that these zeros show as a vanishing minor in the effective mass matrix M_ν. Phenomenological analysis of the case where the zeros of M_R show as a vanishing minor in M_ν for diagonal M_D has been done recently [5, 10]. This, however, is not the most general case. A more general possibility is the simultaneous existence of a texture zero and a vanishing minor in M_ν.

In the present work, we derive the low energy CP-odd WB invariants for neutrino mass matrices with a texture zero and a vanishing minor. In the presence of a texture zero and a vanishing minor, WB invariants provide the simplest tool to investigate whether a specific lepton flavor model leads to leptonic CP violation at low energies. The presence of a texture zero and a vanishing minor, in general, leads to a decrease in the number of independent CP violating phases. In the earlier analysis [6], it was noticed that there are correlations between the Dirac- and Majorana-type CP violating phases. It is, therefore, important to examine the interrelationships between CP-odd WB invariants which are required to vanish as a necessary and sufficient condition for CP conservation. It is the purpose of the present work to examine systematically such interrelationships in terms of the WB invariants constructed from the elements of the neutrino mass matrix.

2 Weak basis invariants from the neutrino mass matrix

A texture zero and a vanishing minor of a neutrino mass matrix in a certain WB may not be present or may appear in different places in other mass matrices obtained by WB transformations so that a neutrino mass matrix with a texture zero and a vanishing minor is not WB invariant. The relevance of CP-odd WB invariants in the analysis of neutrino mass matrices with a texture zero and a vanishing minor is due to the fact that such neutrino mass matrices lead to a decrease in the number of independent CP violating phases. A minimum number of CP-odd WB invariants can be found which will all vanish for CP invariance to hold [11, 14]. A necessary and sufficient condition for low energy CP invariance in the leptonic sector is that the following three WB invariants are identically zero [12]:

$$I_1 = ImgDet[H_\nu, H_l], \tag{2}$$

$$I_2 = ImgTr[H_l M_\nu M_\nu^* M_\nu H_l^* M_\nu^*], \tag{3}$$

$$I_3 = ImgDet[M_\nu^* H_l M_\nu, H_l^*]. \tag{4}$$

Here M_l and M_ν are the mass matrices for the charged leptons and the neutrinos respectively and $H_l = M_l^\dagger M_l$ while $H_\nu = M_\nu^\dagger M_\nu$.

The invariant I_1 was first proposed by Jarlskog [13] as a rephasing invariant measure of Dirac-type CP violation. It, also, describes the CP violation in the leptonic sector and is sensitive to the Dirac-type CP violating phase. The invariants I_2 and I_3 which are the measures of Majorana-type CP violation were first proposed by Branco, Lavoura and Rebelo [14]. The invariant I_3 has a special feature of being sensitive to Majorana-type CP violating phases even in the limit of the exactly degenerate Majorana neutrinos [15].

The CP violation in the lepton number conserving (LNC) processes is contained in the Jarlskog CP invariant J which can be calculated from the WB invariant I_1 using the relation

$$I_1 = -2J(m_e^2 - m_\mu^2)(m_\mu^2 - m_\tau^2)(m_\tau^2 - m_e^2) \times (m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_3^2 - m_1^2) \tag{5}$$

if the neutrino mass matrix M_ν is a complex symmetric matrix with eigenvalues m_1, m_2 and m_3 and the charged lepton mass matrix M_l is diagonal:

$$M_l = diag(m_e, m_\mu, m_\tau). \tag{6}$$

Thus, we have

$$I_1 = -2(m_e^2 - m_\mu^2)(m_\mu^2 - m_\tau^2)(m_\tau^2 - m_e^2) Img(M_{ee}A_{ee} + M_{\mu\mu}A_{\mu\mu} + M_{\tau\tau}A_{\tau\tau}) \quad (7)$$

where the coefficients A_{ee}, $A_{\mu\mu}$ and $A_{\tau\tau}$ are given by

$$A_{ee} = M_{\mu\tau}M_{e\mu}^*M_{e\tau}^*(|M_{\mu\mu}|^2 - |M_{\tau\tau}|^2 - |M_{e\mu}|^2 + |M_{e\tau}|^2) + M_{\mu\mu}M_{e\mu}^{*2} \\ (|M_{e\tau}|^2 - |M_{\mu\tau}|^2) + M_{\mu\mu}^*M_{e\tau}^{*2}M_{\mu\tau}^2, \quad (8)$$

$$A_{\mu\mu} = M_{e\tau}M_{\mu\tau}^*M_{e\mu}^*(|M_{\tau\tau}|^2 - |M_{ee}|^2 - |M_{\mu\tau}|^2 + |M_{e\mu}|^2) + M_{\tau\tau}M_{\mu\tau}^{*2} \\ (|M_{e\mu}|^2 - |M_{e\tau}|^2) + M_{\tau\tau}^*M_{e\mu}^{*2}M_{e\tau}^2, \quad (9)$$

$$A_{\tau\tau} = M_{e\mu}M_{e\tau}^*M_{\mu\tau}^*(|M_{ee}|^2 - |M_{\mu\mu}|^2 - |M_{e\tau}|^2 + |M_{\mu\tau}|^2) + M_{ee}M_{e\tau}^{*2} \\ (|M_{\mu\tau}|^2 - |M_{e\mu}|^2) + M_{ee}^*M_{\mu\tau}^{*2}M_{e\mu}^2, \quad (10)$$

and M_{ij} (i, j=e, μ and τ) are the elements of the neutrino mass matrix in the flavor basis.

Therefore, the Jarlskog CP invariant measure J is given by

$$J = \frac{Img(M_{ee}A_{ee} + M_{\mu\mu}A_{\mu\mu} + M_{\tau\tau}A_{\tau\tau})}{(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_3^2 - m_1^2)}. \quad (11)$$

This relation can be used to calculate J from the mass matrices directly for any lepton mass model rotated to the WB in which M_l is diagonal.

On the other hand, the CP violation in LNV processes can be calculated from the WB invariants I_2 and I_3 given below:

$$I_2 = Img(M_{ee}M_{e\mu}^{*2}M_{\mu\mu}(m_e^2 - m_\mu^2)^2 + M_{\mu\mu}M_{\mu\tau}^{*2}M_{\tau\tau}(m_\mu^2 - m_\tau^2)^2 \\ + M_{\tau\tau}M_{e\tau}^{*2}M_{ee}(m_\tau^2 - m_e^2)^2 + 2M_{ee}M_{e\mu}^*M_{e\tau}^*M_{\mu\tau}(m_e^2 - m_\mu^2)(m_e^2 - m_\tau^2) + 2M_{\mu\mu}M_{\mu\tau}^*M_{e\mu}^*M_{e\tau} \\ (m_\mu^2 - m_\tau^2)(m_\mu^2 - m_e^2) + 2M_{\tau\tau}M_{e\tau}^*M_{\mu\tau}^*M_{e\mu}(m_\tau^2 - m_e^2)(m_\tau^2 - m_\mu^2)) \quad (12)$$

and

$$I_3 = -2(m_e^2 - m_\mu^2)(m_\mu^2 - m_\tau^2)(m_\tau^2 - m_e^2) \times Img(m_e^2M_{ee}B_{ee} + m_\mu^2M_{\mu\mu}B_{\mu\mu} + m_\tau^2M_{\tau\tau}B_{\tau\tau}). \quad (13)$$

where the coefficients B_{ee}, $B_{\mu\mu}$ and $B_{\tau\tau}$ are given by

$$B_{ee} = M_{\mu\tau}M_{e\mu}^*M_{e\tau}^*(m_\mu^4|M_{\mu\mu}|^2 - m_\tau^4|M_{\tau\tau}|^2 - m_e^2m_\mu^2|M_{e\mu}|^2 + m_e^2m_\tau^2|M_{e\tau}|^2) \\ + m_\mu^2M_{\mu\mu}M_{e\mu}^{*2}(m_e^2|M_{e\tau}|^2 - m_\mu^2|M_{\mu\tau}|^2) + m_\mu^2m_\tau^2M_{\mu\mu}^*M_{e\tau}^{*2}M_{\mu\tau}^2, \quad (14)$$

$$B_{\mu\mu} = M_{e\tau}M^*_{\mu\tau}M^*_{e\mu}(m_\tau^4|M_{\tau\tau}|^2 - m_e^4|M_{ee}|^2 - m_\mu^2 m_\tau^2|M_{\mu\tau}|^2 + m_e^2 m_\mu^2|M_{e\mu}|^2) \\ +m_\tau^2 M_{\tau\tau}M^{*2}_{\mu\tau}(m_\mu^2|M_{e\mu}|^2 - m_\tau^2|M_{e\tau}|^2) + m_e^2 m_\tau^2 M^*_{\tau\tau}M^{*2}_{e\mu}M^2_{e\tau}, \quad (15)$$

and

$$B_{\tau\tau} = M_{e\mu}M^*_{e\tau}M^*_{\mu\tau}(m_e^4|M_{ee}|^2 - m_\mu^4|M_{\mu\mu}|^2 - m_e^2 m_\tau^2|M_{e\tau}|^2 + m_\mu^2 m_\tau^2|M_{\mu\tau}|^2) \\ +m_e^2 M_{ee}M^{*2}_{e\tau}(m_\tau^2|M_{\mu\tau}|^2 - m_e^2|M_{e\mu}|^2) + m_e^2 m_\mu^2 M^*_{ee}M^{*2}_{\mu\tau}M^2_{e\mu}. \quad (16)$$

3 Implications for neutrino mass matrices with a texture zero and a vanishing minor

A comprehensive phenomenological analysis of neutrino mass matrices with a texture zero and a vanishing minor has been given in Ref. [6] where it was found that only six out of a total fifteen texture structures are compatible with the current data and have interesting phenomenological implications. In this section, we construct the CP-odd weak basis invariants for these phenomenologically allowed classes listed in Table 1.

Type	Texture Zero	Vanishing Minor
2A	$M_{e\mu}=0$	$M_{\mu\mu}M_{\tau\tau}-M^2_{\mu\tau}=0$
3A.	$M_{e\tau}=0$	$M_{\mu\mu}M_{\tau\tau}-M^2_{\mu\tau}=0$
2D.	$M_{e\mu}=0$	$M_{ee}M_{\tau\tau}-M^2_{e\tau}=0$
3F.	$M_{e\tau}=0$	$M_{ee}M_{\mu\mu}-M^2_{e\mu}=0$
4B.	$M_{\mu\mu}=0$	$M_{e\mu}M_{\tau\tau}-M_{\mu\tau}M_{e\tau}=0$
6C.	$M_{\tau\tau}=0$	$M_{e\mu}M_{\mu\tau}-M_{\mu\mu}M_{e\tau}=0$

Table 1: Six allowed texture structures of M_ν with a texture zero and a vanishing minor.

3.1 Class 2A and 3A

Class 2A has a zero (1,2) element and a vanishing minor corresponding to (1,1) entry. We obtain the invariants by imposing the texture zero condition and the zero minor condition. For class $2A$, the invariants I_1, I_2 and I_3 are given by

$$I_1 = x(|M_{\mu\mu}|^2 + |M_{\mu\tau}|^2)Img(M_{ee}M_{\tau\tau}M^{*2}_{e\tau}), \quad (17)$$

$$I_2 = (m_\tau^2 - m_e^2)^2 Img(M_{ee}M_{\tau\tau}M_{e\tau}^{*2}), \tag{18}$$

$$I_3 = x m_e^2 m_\tau^2 (m_\mu^2 |M_{\mu\mu}|^2 + m_\tau^2 |M_{\mu\tau}|^2) Img(M_{ee}M_{\tau\tau}M_{e\tau}^{*2}). \tag{19}$$

where

$$x = -2(m_e^2 - m_\mu^2)(m_\mu^2 - m_\tau^2)(m_\tau^2 - m_e^2) \tag{20}$$

The WB invariants for class $3A$ can be obtained by interchanging the μ and τ indices in the above relations. It is clear from the above equations that CP violation results from the mismatch in the phases of $M_{e\tau}$, M_{ee} and $M_{\tau\tau}$. A necessary and sufficient condition for the absence of CP violation for class $2A$ is given by

$$2arg(M_{e\tau}) = arg(M_{ee}) + arg(M_{\tau\tau}). \tag{21}$$

However, it should be noted that this is not the unique way to express the condition of CP invariance as the condition of the vanishing minor gives us some extra freedom to express this condition in a different way which of course is equivalent to the above condition. The condition given above is equivalent to the condition that the neutrino mass matrix M_ν can be factorized as $PM_\nu^r P$ where P is a diagonal phase matrix diag($e^{i\phi_1}$, $e^{i\phi_2}$, $e^{i\phi_3}$) and M_ν^r is a real matrix. Therefore, the necessary and sufficient condition for CP conservation is that the neutrino mass matrix M_ν can be written as

$$M_\nu = PM_\nu^r P. \tag{22}$$

However, it is not necessary that neutrino mass matrices with a texture zero and a vanishing minor always satisfy this equation.

3.2 Class 2D and 3F

Neutrino mass matrices belonging to class $2D$ have a texture zero at (1,2) position and a vanishing minor corresponding to (2,2) element. For class $2D$, the invariants I_1, I_2 and I_3 are given by

$$I_1 = -x\left(1 + \frac{|M_{e\tau}|^2}{|M_{ee}|^2}\right) Img(M_{\mu\mu}M_{ee}^{*}M_{\mu\tau}^{*2}M_{e\tau}^2), \tag{23}$$

$$I_2 = \frac{(m_\mu^2 - m_\tau^2)^2}{|M_{ee}|^2} Img(M_{\mu\mu}M_{ee}^{*}M_{\mu\tau}^{*2}M_{e\tau}^2), \tag{24}$$

$$I_3 = -x\left(m_e^2 m_\mu^2 m_\tau^2 + m_\mu^2 m_\tau^4 \frac{|M_{e\tau}|^2}{|M_{ee}|^2}\right) Img(M_{\mu\mu}M_{ee}^{*}M_{\mu\tau}^{*2}M_{e\tau}^2). \tag{25}$$

The necessary and sufficient condition of CP invariance for this class is given by

$$arg(M_{ee}) + 2arg(M_{\mu\tau}) = arg(M_{\mu\mu}) + 2arg(M_{e\tau}). \tag{26}$$

The WB invariants for class $3F$ can be obtained by interchanging the μ and τ indices in the above relations.

3.3 Class 4B and 6C

The neutrino mass matrices belonging to class 4B have a texture zero at (2,2) position and a vanishing minor corresponding to (1,2) element. The weak basis invariants for this class are given by

$$I_1 = -x(1 + \frac{|M_{e\mu}|^2}{|M_{\mu\tau}|^2})Img(M_{ee}M^*_{\tau\tau}M^{*2}_{e\mu}M^2_{\mu\tau}), \tag{27}$$

$$I_2 = (\frac{|M_{e\tau}|^2(m_\tau^2 - m_e^2)^2}{|M_{e\mu}|^2|M_{\mu\tau}|^2} + \frac{2(m_e^2 - m_\mu^2)(m_e^2 - m_\tau^2)}{|M_{\mu\tau}|^2})Img(M_{ee}M^*_{\tau\tau}M^{*2}_{e\mu}M^2_{\mu\tau}), \tag{28}$$

$$I_3 = -x(m_e^2 m_\mu^2 m_\tau^2 + m_\mu^2 m_e^4\frac{|M_{e\mu}|^2}{|M_{\mu\tau}|^2})Img(M_{ee}M^*_{\tau\tau}M^{*2}_{e\mu}M^2_{\mu\tau}). \tag{29}$$

It is clear from these equations for I_1, I_2 and I_3 that the neutrino mass matrices of class $4B$ will be CP invariant if the phases of the mass matrix are fine tuned to satisfy the condition

$$arg(M_{ee}) + 2arg(M_{\mu\tau}) = arg(M_{\tau\tau}) + 2arg(M_{e\mu}). \tag{30}$$

The weak basis invariants for class $6C$ can be obtained by interchanging the μ and τ indices in the above invariants. The conditions of CP invariance for neutrino mass matrices belonging to different viable classes have been summarized in Table 2.

Class	CP invariance condition
2A	$2\ arg(M_{e\tau}) = arg(M_{ee}) + arg(M_{\tau\tau})$
2D	$arg(M_{ee}) + 2\ arg(M_{\mu\tau}) = arg(M_{\mu\mu}) + 2\ arg(M_{e\tau})$
4B	$arg(M_{ee}) + 2\ arg(M_{\mu\tau}) = arg(M_{\tau\tau}) + 2\ arg(M_{e\mu})$

Table 2: Conditions for CP invariance for some viable classes of neutrino mass matrices with a texture zero and a vanishing minor

The three WB invariants I_1, I_2 and I_3 are related to each other for all viable classes

of neutrino mass matrices. The relations are summarized in Table 3. The Dirac-type phase δ contributing to CP violation in LNC processes is contained in I_1 while the invariants I_2 and I_3 are measures of Majorana-type CP violation which contributes to LNV processes. However, the interrelationships between I_1, I_2 and I_3 for various viable classes of neutrino mass matrices suggest that the three CP violating phases are not independent and there is only one independent physical phase in all the phenomenologically viable neutrino mass matrices with a texture zero and a vanishing minor which contributes to CP violation in both LNC and LNV processes. Hence, the distinction between Dirac- and Majorana-type phases cannot be maintained in the neutrino mass matrices with a texture zero and a vanishing minor.

Class	$\frac{I_1}{I_2}$	$\frac{I_1}{I_3}$
2A	$\frac{2(m_e^2-m_\mu^2)(m_\mu^2-m_\tau^2)(\lvert M_{\mu\mu}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(m_e^2-m_\tau^2)}$	$\frac{(\lvert M_{\mu\mu}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(m_e^2 m_\mu^2 m_\tau^2\lvert M_{\mu\mu}\rvert^2+m_e^2 m_\tau^4\lvert M_{\mu\tau}\rvert^2)}$
3A	$\frac{2(m_e^2-m_\tau^2)(m_\mu^2-m_\tau^2)(\lvert M_{\tau\tau}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(m_\mu^2-m_e^2)}$	$\frac{(\lvert M_{\tau\tau}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(m_e^2 m_\mu^2 m_\tau^2\lvert M_{\tau\tau}\rvert^2+m_e^2 m_\mu^4\lvert M_{\mu\tau}\rvert^2)}$
2D	$\frac{2(m_e^2-m_\mu^2)(m_\tau^2-m_e^2)(\lvert M_{ee}\rvert^2+\lvert M_{e\tau}\rvert^2)}{(m_\mu^2-m_\tau^2)}$	$\frac{(\lvert M_{ee}\rvert^2+\lvert M_{e\tau}\rvert^2)}{(m_e^2 m_\mu^2 m_\tau^2\lvert M_{ee}\rvert^2+m_\mu^2 m_\tau^4\lvert M_{e\tau}\rvert^2)}$
3F	$\frac{2(m_e^2-m_\mu^2)(m_\tau^2-m_e^2)(\lvert M_{ee}\rvert^2+\lvert M_{e\mu}\rvert^2)}{(m_\tau^2-m_\mu^2)}$	$\frac{(\lvert M_{ee}\rvert^2+\lvert M_{e\mu}\rvert^2)}{(m_e^2 m_\mu^2 m_\tau^2\lvert M_{ee}\rvert^2+m_\tau^2 m_\mu^4\lvert M_{e\mu}\rvert^2)}$
4B	$\frac{2(m_e^2-m_\mu^2)(m_\mu^2-m_\tau^2)(\lvert M_{e\mu}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(\frac{\lvert M_{e\tau}\rvert^2(m_\tau^2-m_e^2)}{\lvert M_{e\mu}\rvert^2}+2(m_\mu^2-m_e^2))}$	$\frac{(\lvert M_{e\mu}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(m_e^2 m_\mu^2 m_\tau^2\lvert M_{\mu\tau}\rvert^2+m_\mu^2 m_e^4\lvert M_{e\mu}\rvert^2)}$
6C	$\frac{2(m_e^2-m_\tau^2)(m_\mu^2-m_\tau^2)(\lvert M_{e\tau}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(\frac{\lvert M_{e\mu}\rvert^2(m_e^2-m_\mu^2)}{\lvert M_{e\tau}\rvert^2}+2(m_e^2-m_\tau^2))}$	$\frac{(\lvert M_{e\tau}\rvert^2+\lvert M_{\mu\tau}\rvert^2)}{(m_e^2 m_\mu^2 m_\tau^2\lvert M_{\mu\tau}\rvert^2+m_\tau^2 m_e^4\lvert M_{e\tau}\rvert^2)}$

Table 3: Ratios $\frac{I_1}{I_2}$ and $\frac{I_1}{I_3}$ for all viable classes of neutrino mass matrices with a texture zero and a vanishing minor.

4 Conclusions

We have calculated the CP-odd weak basis invariants for LNC and LNV processes for all the phenomenologically viable neutrino mass matrices with a texture zero and a vanishing minor in the flavor basis. All these invariants must vanish for CP to be conserved. We find that neutrino mass matrices with a texture zero and a vanishing minor are, in general, CP violating unless their phases are fine tuned. The three WB invariants in each class are found to be related to each other so that all viable classes

of neutrino mass matrices with a texture zero and a vanishing minor have only one independent physical phase which contributes to CP violation in both LNC and LNV processes, and, hence, cannot be labelled as either Dirac or Majorana.

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